Math 103 Day 18: Areas and Definite Integrals

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Outline

1 Area Under a Curve

Useful Formulas

$$1 + 2 + 3 + ... + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + ... + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + ... + n^3 = \sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$$

n	L_n	R_n
10	.285	.385
20	.308	.358
30	.316	.350
50	.323	.343
100	.328	.338
1000	.333	.334

Definition

The **area** A of a region S that lies under the graph of a continuous function f is the limit of the sum of areas of the approximating rectangles:

$$A = lim_{n\to\infty} R_n = lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x]$$

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Let x_i^* be any value between x_{i-1} and x_i. A collection of such points are called **sample points**. Then

$$A = lim_{n\to\infty}[f(x_1^*)\Delta x + f(x_2^*)\Delta x + ... + f(x_n^*)\Delta x]$$

Definition

(**Definite Integral**)If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$. We let $x_0(= a), x_1, x_2, ..., x_n(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, ..., x_n^*$ be any **sample points** in these subintervals. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)dx = lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that this limit exists. If it does exist, we say that f is **integrable** on $[a, b]$.

Theorem

If f is continuous on $[a, b]$, or if f has only a finite number of jump discontinuities, then f is integrable on $[a, b]$; that is , the definite integral $\int_a^b f(x)dx$ exists.

Theorem

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx = lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.